INFLUENCE OF MASS-TRANSFER PROCESSES ON THE NONLINEAR FILTRATION OF MULTICOMPONENT SYSTEMS

R. M. Sattarov and I. A. Vazhnova

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A solution of the nonlinear filtration equations of multicomponent systems is proposed in the form of steady waves and their development over time is analyzed as a function of the character of mass-transfer processes.

In designing gas, gas–condensate, and gas–condensate–petroleum fields, one of the basic problems is to choose well-founded technological parameters of the working. This involves solving gas-hydrodynamic problems, taking account of the characteristic phenomena occurring in the bed.

For gas–condensate fileds, mass-transfer (sorption, condensation, evaporation) between the porous medium and the gas–condensate system and also between the individual components of the gas–condensate system are characteristic. In this connection, the mass-continuity equation is written in the form

$$\operatorname{div}(\rho\bar{v}) + \frac{\partial}{\partial t}(\rho m) = f. \tag{1}$$

Since in developing a field the temperature there changes only slightly in comparison with the pressure, all the quantities appearing in Eq. (1) may be regarded as depending only on the pressure. Mass-transfer processes are related to the pressure variation in a complex manner. Thus, at the beginning of the working, decrease in pressure is associated with deposition of condensate from the gas phase to the porous medium; with further decrease in pressure, the condensate begins to evaporate. The gas-phase mass at first begins to increase with evaporation and then decreases on account of processes of retrograde condensation. The period of pressure reduction is characterized by desorption. The amount of desorbed gas and evaporating condensate increases the gas-hydrodynamic characteristics of the filtrational flow, and the amount of condensate deposited decreases. With increase in pressure, the filtration process is associated with gas-phase absorption on the rock framework and direct condensation, which decrease the gasdynamic characteristics of the filtrational flux, as well as processes of retrograde evaporation, which increase these characteristics. Taking account of the foregoing, the mass-transfer function f may be written in the form

$$f(P) = q_{\mathrm{u}}(P) - q_{\mathrm{d}}(P).$$

Mass transfer between the gas–condensate system and the porous medium occurs especially intensely in argillized collectors, which have good sorptional properties. The amount of gas sorbed there may reach 10% of the amount of gas in the pores. The property of creep is intrinsic to argillized collectors. This leads to disrutpion of the equilibrium relation between the filtration rate and the pressure gradient. In [1], an integral transformation was proposed for the description of these phenomena. Then

$$\bar{v} = -\frac{k}{\mu}\int_0^t F(t-\tau)\,\mathrm{grad}\,P(\tau)\,d\tau, \tag{2}$$

where F is the kernel of the integral transformation. It is determined from experimental data for the various collectors. However, since determining the form of the kernel involves differntiating experimentally determined functions, it was proposed in [2] that the form of

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the kernel be specified analytically, and the parameters appearing there be determined experimentally. On this basis, the kernel is specified in the form

$$F(t, \tau) = \exp[-(t-\tau)/\theta], \tag{3}$$

where θ is the characteristic relaxation time of the clay. Then, taking account of Eq. (3), Eq. (2) is transformed to give

$$\bar{v} + \theta \frac{\partial \bar{v}}{\partial t} = -\frac{k}{\mu} \operatorname{grad} P. \tag{4}$$

Consider the features of nonlinear filtration of gas–condensate systems, taking account of mass-transfer processes.

Neglecting the influence of liquid condensate on the effective porosity of the system, and assuming that the porosity m and density ρ depend only on the pressure, these quantities are expanded in Taylor series in the vicinity of the steady state (m_0, ρ_0, P_0), retaining terms of second-order smallness

$$m = m_0 \left[1 + \beta_m (P - P_0) + \frac{\beta_m^2}{2} (P - P_0)^2\right],$$
$$\rho = \rho_0 \left[1 + \beta_\rho (P - P_0) + \frac{\beta_\rho^2}{2} (P - P_0)^2\right]. \tag{5}$$

Then

$$\frac{\partial}{\partial t} (\rho m) = m_0 \rho_0 (\beta_\rho + \beta_m) \left[1 + (\beta_\rho + \beta_m)(P - P_0)\right] \frac{\partial P}{\partial t}.$$

It is expedient to introduce the functions

$$M = \rho \bar{v}, \quad \delta(P) = m_0 \rho_0 (\beta_\rho + \beta_m)[1 + (\beta_\rho + \beta_m)(P - P_0)]. \tag{6}$$

Assuming that filtration is one-dimensional, and taking account of Eqs. (5) and (6), the system in Eqs. (1) and (4) is written as follows

$$M + \theta \frac{\partial M}{\partial t} = -\frac{k_0 \rho_0}{\mu_0} \frac{\partial P}{\partial x},$$
$$\frac{\partial M}{\partial x} + \delta(P) \frac{\partial P}{\partial t} = f(P). \tag{7}$$

The solution of Eq. (7) will be sought in the class of steady traveling waves in which all the dependent variables are a function only of $\xi = x - ut$, where u is the velocity of wave propagation. In the new coordinate system, Eq. (7) takes the form

$$\frac{dM}{d\xi} = f(P) + u\delta(P) \frac{dP}{d\xi},$$
$$\frac{dP}{d\xi} = -\varkappa M + \theta \varkappa u \frac{dM}{d\xi} \tag{8}$$

or

$$\frac{dM}{d\xi} = \frac{-f(P) + u\varkappa\delta(P) M}{1 - u^2 \varkappa \delta(P) \theta} = \varphi_1(P, M),$$

$$\frac{dP}{d\xi} = -\frac{\varkappa M - u\varkappa\theta f(P)}{1 - u^2\varkappa\delta(P)\theta} = \varphi_2(P, M), \qquad (9)$$

where $\varkappa = \mu_0/(k_0\rho_0)$. Qualitative analysis of Eq. (9) may be performed in the phase plane (P, M). Expanding the right-hand side of Eq. (9) in Taylor series in the vicinity of singular points, while retaining first order terms, and taking into account that $dP/d\xi = dM/d\xi = 0$ at the singular points, or $-f(P_0) + u\varkappa\delta(P_0)M_0 = 0$ and $\varkappa M_0 - u\varkappa\theta f(P_0) = 0$, Eq. (9) may be replaced by the relation

$$\frac{dP}{d\xi} = aP + bM, \quad \frac{dM}{d\xi} = cP + dM, \qquad (10)$$

where

$$a = \frac{\partial\varphi_2(0)}{\partial P} = \frac{u\varkappa\theta f'_P(0)}{1 - u^2\varkappa\theta\delta(0)}; \quad b = \frac{\partial\varphi_2(0)}{\partial M} = -\frac{\varkappa}{1 - u^2\varkappa\theta\delta(0)};$$
$$c = \frac{\partial\varphi_1(0)}{\partial P} = \frac{f'_P(0)}{1 - u^2\varkappa\theta\delta(0)}; \quad d = \frac{\partial\varphi_1(0)}{\partial M} = -\frac{u\varkappa\delta(0)}{1 - u^2\varkappa\theta\delta(0)}.$$

Here and below, P and M denote not the pressure and mass flow rate but their deviations relative to the singular points.

The characteristic equation of Eq. (10) is

$$\gamma^2 - (a + d)\gamma + (ad - bc) = 0.$$

The equilibrium state of the system will be stable if [3], $a + d < 0$ or $(-u\varkappa\theta f'_P + u\varkappa\delta(0))/(1 - u^2\varkappa\theta\delta(0)) > 0$; hence, when $u < \sqrt{1/\theta\varkappa\delta}$

$$f'_P < \delta/\theta. \qquad (11)$$

All the trajectories on the phase plane correspond to some steady wave. However, since solutions that are limited in amplitude are of interest from physical considerations, the conditions under which the singular points are centers (corresponding to closed trajectories on the phase plane and waves periodic in the coordinate ξ) or saddles (when the trajectories on the phase plane correspond to the propagation of a perturbation shifting the system from one equilibrium state to another) are analyzed.

The condition of existence of periodic solutions is that the real part of the roots be zero (under the condition that the roots are complex), i.e., $a + d = 0$, $(a + d)^2/4 < (ad - bc)$ when $ad - bc > 0$. Then from Eq. (10)

$$[(u^2\varkappa^2\theta\delta f'_P - \varkappa f'_P)/(1 - \theta\varkappa\delta u^2)^2] < 0$$

or

$$[\varkappa f'_P/(1 - \varkappa\theta\delta u^2)] > 0.$$

When

$$f'_P > 0 \qquad (12)$$

periodic oscillations with a limited propagation velocity $u < 1/\sqrt{\theta\varkappa\delta}$ exist. The condition of existence of a saddle is $ad - bc < 0$ or

$$f'_P < 0. \qquad (13)$$

The region of pressure variation in which the behavior of the system is qualitatively changed is established from analysis of Eqs. (11)-(13). With reduction in pressure, the

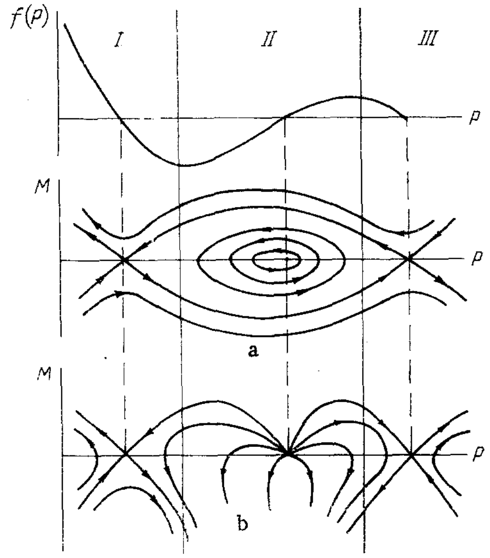

Fig. 1. Possible form of the pressure dependence of the mass transfer and the corresponding trajectory on the phase plane (P, M): I, III) regions where $f'_P < 0$; II) region where $0 < f'_P < \delta/\theta$ (a) and $f'_P > \delta/\theta > 0$ (b).

condition $f'_P > \delta/\theta$ corresponds to greater intensity of desorption in comparison with condensation; the rate of desorption is higher than the rate of condensation by an amount greater than δ/θ. In this case, any small perturbations are amplified: the system is unstable (Fig. 1, region IIb).

In the pressure range where the desorption rate is higher than the condensation rate by an amount less than δ/θ periodic vibrations in the system propagating at limited velocity are possible (Fig. 1, region IIa). In the case when the condensation processes occur at great intensity, i.e., $f'_P < 0$, the perturbations arising in the system are either extinguished if there is only one equilibrium state of the system or else transfer the system to another equilibrium state (Fig. 1, regions I, III).

Note that, in investigating the system using the phase plane, only the character of the steady waves may be established; their development over time and space cannot be investigated. These characteristics are investigated by the method of analysis of transient processes developed in [4]. In this case, the initial system in Eqs. (1) and (4) is reduced to the following form

$$\left(1+\theta\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}(\rho m)=\frac{\rho_0 k_0}{\mu_0}\frac{\partial^2 P}{\partial x^2}+\left(1+\theta\frac{\partial}{\partial t}\right)f(P). \tag{14}$$

Approximating q_u and q_d by linear dependences

$$q_u=\alpha_u(P-P_0-P_u),\quad q_d=\alpha_d(P-P_0-P_d),\quad f(P)=q_u-q_d$$

and taking account of the pressure dependence of m and ρ in the form in Eq. (5), Eq. (14) is replaced by the relation

$$\left(1+\theta\frac{\partial}{\partial t}\right)\frac{\partial}{\partial t}(AP+BP^2)=\frac{\rho_0 k_0}{\mu_0}\frac{\partial^2 P}{\partial x^2}+\alpha\left(1+\theta\frac{\partial}{\partial t}\right)P+D,$$

where

$$P\to P'=P-P_0;\ A=\frac{m_0}{c^2};\ B=\frac{\rho_0 m_0}{2}(\beta_m+\beta_\rho)^2;\ 1/c^2=\rho_0(\beta_\rho+\beta_m);$$

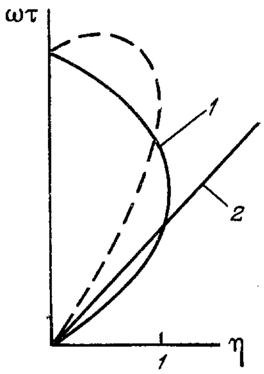

Fig. 2. Graphical analysis of the dependence $\omega\tau = \arcsin \eta = K\eta$ (dashed curve); 1) $\sin^{-1} \eta$; 2) $K\eta$.

$$\alpha = \alpha_u - \alpha_d;\ D = \alpha_d P_d - \alpha_u P_u,$$

or

$$A_1 \left(1 + \theta \frac{\partial}{\partial t}\right) \frac{\partial}{\partial t} (P + B_1 P^2) = \frac{\partial^2 P}{\partial x^2} + \frac{\alpha\mu_0}{\rho_0 k_0} \left(1 + \theta \frac{\partial}{\partial t}\right) P + D_1, \tag{15}$$

where

$$A_1 = m_0\mu_0/c^2 k_0\rho_0;\ B_1 = \mu_0/2\rho_0^2 c^2 k_0;\ D_1 = D\mu_0/k_0\rho_0.$$

Taking into account that the characteristic relaxation time of the clay θ much exceeds the hydrodynamic time, Eq. (15) is simplified

$$A_1\theta \frac{\partial^2}{\partial t^2} (P + B_1 P^2) - \frac{\partial^2 P}{\partial x^2} - \theta \frac{\mu_0 \alpha}{k_0 \rho_0} \frac{\partial P}{\partial t} = 0. \tag{16}$$

In the absence of nonlinearity ($B_1 = 0$) and mass transfer ($\alpha = 0$), the solution of Eq. (16) will be steady waves of constant profile. If the perturbation $P = F(t)$ is specified on $x = 0$, the solution of Eq. (16) is

$$P(x, t) = F(t - x\sqrt{A_1\theta}). \tag{17}$$

It is natural to assume that, if nonlinearity and mass transfer are present but small, i.e., $B_1 P \ll 1$, $\alpha \ll 1$, the solution locally takes the form in Eq. (17) but slowly changes its form as it moves along x.

Applying the coordinate transformation $\xi = x$, $\tau = t - x\sqrt{A_1\theta}$ to Eq. (16), and taking into account that

$$\frac{\partial P}{\partial x} = \frac{\partial P}{\partial \xi} + \frac{\partial P}{\partial \tau} \frac{\partial \tau}{\partial x}$$

it follows from Eq. (16) that

$$A_1 B_1 \theta \frac{\partial^2 P^2}{\partial \tau^2} - \frac{\partial^2 P}{\partial \xi^2} + 2\sqrt{A_1\theta} \frac{\partial^2 P}{\partial \tau \partial \xi} - \theta \frac{\mu_0 \alpha}{k_0 \rho_0} \frac{\partial P}{\partial \tau} \Big] = 0. \tag{18}$$

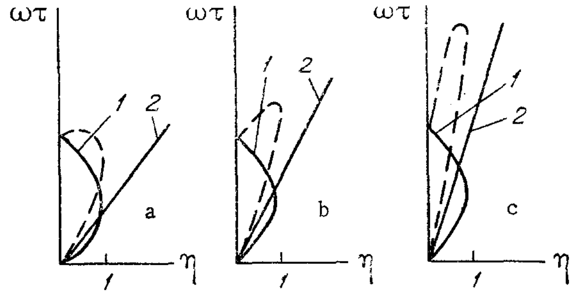

Fig. 3. Change in amplitude of initial perturbation A_0 in the absence of mass transfer ($\delta_2 = 0$). Notation as in Fig. 2: a) $\xi_0 = \xi_{cr}$, $A = A_0$; b) $\xi_1 > \xi_{cr}$, $A_1 < A_0$; c) $\xi_2 > \xi_1$, $A_2 < A_1$.

Since variation in form of the wave along ξ is assumed to be small, then $\partial^2 P/\partial\xi^2$ may be neglected. Then

$$\frac{\partial}{\partial\tau}\left[2A_1B_1\theta\left(P\frac{\partial P}{\partial\tau}\right)+2\sqrt{A_1\theta}\frac{\partial P}{\partial\xi}-\theta\frac{\mu_0\alpha}{k_0\rho_0}\frac{\partial P}{\partial\tau}\right]=0.$$

Integration with respect to τ gives

$$\frac{\partial P}{\partial\xi}+\delta_1 P\frac{\partial P}{\partial\tau}+\delta_2 P=F(\xi), \tag{19}$$

where $F(\xi)$ is an arbitrary function. If $P(t = 0) = 0$, then $F(\xi) \equiv 0$

$$\delta_1=\sqrt{A_1\theta}\,B_1,\quad \delta_2=\frac{1}{2}\frac{\mu_0(\alpha_d-\alpha_u)}{k_0\rho_0}\sqrt{\frac{\theta}{A_1}}. \tag{20}$$

In implicit form, the solution of Eq. (19) when $F(\xi) \equiv 0$ is

$$\tau=\varphi^{-1}[P\exp(\delta_2\xi)]+(\delta_1/\delta_2)P[\exp(\delta_2\xi)-1],$$

where $\varphi(t)$ is the pressure at $x = 0$.

Assuming that $P(x = 0) = P_0 \sin\omega t$, the solution of the problem may be written in the following dimensionless form

$$\omega\tau=\arcsin\left[\frac{P}{P_0}\exp(\delta_2\xi)\right]+\frac{\delta_1}{\delta_2}\omega P_0[1-\exp(-\delta_2\xi)]\frac{P}{P_0}\exp(\delta_2\xi). \tag{21}$$

It may be analyzed graphically in the coordinates $(\omega\tau, \eta)$:

$$\eta=\frac{P}{P_0}\exp(\delta_2\xi),\quad K=\frac{\delta_1\omega P_0}{\delta_2}[1-\exp(-\delta_2\xi)], \tag{22}$$

$$\omega\tau=\arcsin\eta+K\eta.$$

As is evident from Fig. 2, the function η and hence also P become multivalued when $K \geqslant 1$ i.e., a shock wave is formed. If there is no mass transfer ($\delta_2 = 0$), the solution takes the form

$$\omega\tau=\arcsin\left(\frac{P}{P_0}\right)+\delta_1\omega P_0\xi\left(\frac{P}{P_0}\right).$$

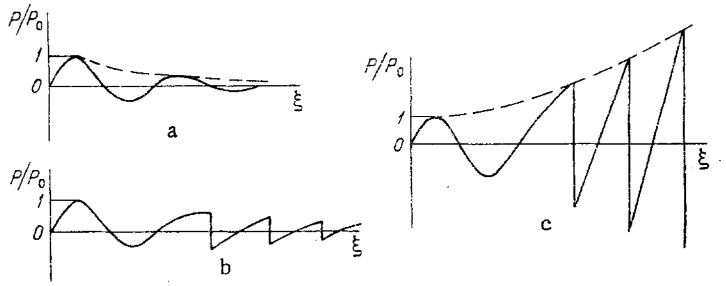

Fig. 4. Variation in form of initial perturbation: a) when $\delta_2 > 0$, $P_0 < P_{cr}$; b) when $\delta_2 > 0$, $P_0 \geqslant P_{cr}$; c) when $\delta_2 < 0$; dahsed curve shows the envelope of the amplitude of variation in P/P_0.

The condition for the appearance of a shock wave is $K \geqslant 1$, i.e., $\delta_1 \omega P_0 \xi \geqslant 1$. Hence the distance $\xi_{cr} \geqslant 1/\delta_1 \omega P_0$, at which a shock wave is formed may be determined; when $\xi > \xi_{cr}$, its amplitude decreases and it becomes serrated (Fig. 3).

In the presence of both nonlinearity (δ_1 0) and mass transfer ($\delta_2 \neq 0$), the appearance and character of shock-wave propagation wave depends on the intensity ratio of the occurrence of mass-transfer processes. Assume that $\alpha_d > \alpha_u$, i.e., $\delta_2 > 0$.

The condition of shock-wave formation, as is evident from Eq. (22), is

$$\frac{\delta_1 \omega P_0}{\delta_2} [1 - \exp(-\delta_2 \xi)] \geqslant 1. \tag{23}$$

It is obvious that, when $\delta_1 \omega P_0/\delta_2 < 1$, i.e., when $P_0 < \delta_2/\delta_1 \omega$, the shock wave is not formed at at any distance ξ as large as may be desired. Hence, the condition of shock-wave formation is $P_0 \geqslant [\delta_2/\delta_1 \omega] = P_{cr}$.

Characteristic variations of the wave when $\delta_2 > 0$, $P_0 < P_{cr}$, $P_0 \geqslant P_{cr}$ are shown in Fig. 4a, b. The critical value P_{cr} increases as the loss of condensate increases and as the nonlinearity coefficient decreases. The value of the critical cross section at which shock-wave formation occurs is determined from Eq. (23). If $\alpha_d < \alpha_u$, shock-wave appearance at a distance ξ_{cr} determined from Eq. (23) is possible with any small perturbation P_0. Its amplitude increases along ξ according to the law $P_0 \exp(|\delta_2|\xi)$ (Fig. 4c), i.e., in conditions where desorption processes are more intense than condensation processes (in conditions of depletion), the system becomes unstable and any small perturbations are amplified.

What is the possible propagation velocity of the perturbation? Assume that the compressibility factor of argillaceous sandstone $\beta_m = 1.5 \cdot 10^{-10}$ Pa^{-1}; the compressibility factor of the condensate $\beta_\rho = 14 \cdot 10^{-10}$ Pa^{-1}; the density of the condensate $\rho_0 = 1$ kg/m^3; the porosity $m_0 = 0.1$ the viscosity $\mu_0 = 2 \cdot 10^{-5}$ Pa·sec. For various argillized collectors, the permeability k_0 usually varies in the range 10^{-13}–10^{-14} m^2. The relaxation time determined by the creep of the argillized rock oscillates over broad limits 10^3–10^7 sec [5, 6]. Then $u = (A_1 \theta)^{-\frac{1}{2}} = [m_0 \mu_0 (\beta_\rho + \beta_m) \theta k_0^{-1}]^{-\frac{1}{2}} \approx 2 \cdot 10^{-1} - 2 \cdot 10^{-3}$.

NOTATION

P, pressure (P_0, initial pressure in bed); ρ, density of filtrational flux (ρ_0, density at pressure P_0); $\bar{v}$, filtration rate; μ, viscosity of fluid (μ_0, viscosity at pressure P_0); m, porosity of collectors (m_0, porosity at pressure P_0); f(P), pressure-dependent mass-transfer function; q_u, q_d, mass transfer in unit volume per unit time, respectively, increasing and decreasing the filtrational flux; k, permittivity of collector (k_0, permittivity at pressure P_0); $F(t, \tau)$, kernel of integral transformation; θ, relaxation time of argillized collector; t, τ, time; x, spatial coordinate; M, mass velocity; u, propagation velocity of perturbation; wave; α_u, α_d, intensity of mass-transfer processes.

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GREEN'S-FUNCTION METHOD FOR SOLVING PROBLEMS OF NONEQUILIBRIUM ADSORPTION AND CONVECTIVE DIFFUSION OF IMPURITY IN A MEDIUM

Yu. N. Gordeev and N. A. Kudryashov UDC 532.546

The Green's function method is used to solve problems of impurity transfer by a carrier-gas flow in a semiinfinite medium, taking account of convective diffusion, nonequilibrium adsorption, and radioactive decay.

In describing the propagation of adsorbed impurity in a porous medium under the action of a carrier gas, as a rule, account is taken of longitudinal diffusion and mass transfer from a gas flow to the adsorbent granule. The convective-diffusion coefficient depends on the velocity of carrier-gas motion and the characteristic dimension of the porous medium $D = D_0 + \Delta\nu$ [1]. Hence it follows that, for a homogeneous porous medium and a constant gas-flow velocity, the convective diffusion coefficient is a constant and does not depend on the coordinates and the time. The characteristic length of the porous layer, beginning with which convective diffusion significantly influence the impurity characteristics, is determined from the estimate $l \geqslant \sqrt{Dt_0}$, although in reality the impurity "front" may be distorted on account of diffusional blurring at relatively small distances.

Impurity adsorption is divided into three stages [2]: external mass transfer, the act of adsorption, and internal diffusion in adsorbent grains. The second stage usually occurs considerably more rapidly than the other two.

External mass transfer occurs by molecular diffusion to the surface and mixing of impurity in the flow and is characterized by a kinetic adsorption coefficient β, which is related to the flow velocity and grain size by the dimensionless equation [2] $Nu = A\,Re^n Pr^m$. For a homogeneous porous medium and at constant gas-flow velocity, the kinetic adsorption coefficient will also be constant.

The adsorption kinetics must be taken into account when $t \sim \beta^{-1}$, i.e., when the characteristic time of the process is comparable with the inverse of the kinetic coefficient.

If the characteristic grain size of the porous medium satisfies the condition $d \ll \sqrt{D_0 t_0}$, the propagation of adsorbed impurity in the porous medium when $d^2 D_0^{-1} \ll t_0 \lesssim l^2 D^{-1}$ is described by the following system of equations

$$u_t + a_t + \nu u_x + \lambda(u + a) = Du_{xx}, \quad (1)$$

$$a_t = \beta(u - u^*) - \lambda a, \quad u^* = \gamma a. \quad (2)$$

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